The Model-based Approach to Computer-aided Medical Decision Support

Lecture 4: Causal Independence

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Introduction

- Clinical decision support, because . . .
  - doctors make more mistakes than you would accept (as a patient)
  - some of their actions are harmful

- Deployment of:
  - probabilistic graphical models
  - logical methods
  - combinations

- Causal modelling for the management of infectious disease (work together with Stefan Visscher) and detection of breast cancer (work with Marina Velikova)
Problem

- ICU at Utrecht MC
- Diagnosis and antimicrobial treatment of patients with ventilator-associated pneumonia (VAP)
- About 15-20% of ICU patients develop VAP
- Mortality rate: up to 40%
- Up to 50% of antibiotics in ICUs are prescribed for airway infections
Software Infrastructure

- PHP Module
- Apache HTTP Server
- Web Browser

Relations:
- SQL
- Data
- Variable-value pairs
- Variable-value-probability triples
- Bayesian Network

CPR

Reasoning System
Global Model Pneumonia

- hospitalisation
- colonisation
- aspiration
- mechanical ventilation
- immunological status
- symptoms signs, lab
- side effects
- antimicrobial therapy
- organism susceptibility
- coverage

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Detailed Pneumonia Network

- hospitalisation
- colonisation PA
- colonisation HI
- colonisation SP
- colonisation
- pneumonia PA
- pneumonia HI
- pneumonia SP
- pneumonia
- symptoms
- signs
- lab
- Antibiotics
- coverage
- overall coverage
- aspiration
- mechanical ventilation
- immunological status
- overall coverage
- coverage
Prediction

\[ \Pr(\text{pneumonia}) = 1.0 \quad \text{or} \quad \Pr(\text{pneumonia}) = ? \]
Specification of Interactions

- Compact specification: probability tables
  \[ P(X_i \mid \text{pa}(X_i)) \]

  can still be large even when taking into account independence information

- Easy way to map domain knowledge to entries into a probability table

- Way to use qualitative knowledge about interactions as constraints to probabilistic information

- Might be useful in developing applications
People become colonised by bacteria when entering a hospital, which may give rise to pneumonia.
Bayesian-network Modelling

Qualitative

causal modelling

Quantitative

interaction modelling

Cause → Effect

\[ P(\text{Inf} \mid \text{BR}_A, \text{BR}_B, \text{BR}_C) \]
Causal Independence

\[ P(e \mid C_1, \ldots, C_n) = \sum_{I_1, \ldots, I_n} P(e \mid I_1, \ldots, I_n) \]

\[ \times \prod_{k=1}^{n} P(I_k \mid C_k) = \sum_{f(I_1, \ldots, I_n) = e} \prod_{k=1}^{n} P(I_k \mid C_k) \]

Note: \( P(i_k \mid \bar{c}_k) = 0 \) – absent causes don’t contribute
Boolean Interaction

Commutative, associative: $\wedge, \vee, \leftrightarrow, \emptyset, \top, \bot$

Commutative, non-associative: $\downarrow, |$

Non-commutative, associative: $p_1, p_2, n_1, n_2$

Non-commutative, non-associative: $\rightarrow, \leftarrow, <, >$
Symmetric Boolean Functions

Order of arguments doesn’t matter; defined in terms of exact function $e_k$:

$$f(I_1, \ldots, I_n) = \bigvee_{k=0}^{n} e_k(I_1, \ldots, I_n) \land \gamma_k$$

where $\gamma_k$ are Boolean constants only dependent of the function $f$

Example: threshold function $\tau_l$:

$$\tau_l(I_1, \ldots, I_n) = \bigvee_{k=l}^{n} e_k(I_1, \ldots, I_n)$$
Decomposition by Counting

Threshold function $\tau_3$:

![Diagram showing the decomposition process with threshold function $\tau_3$.]
By antibiotic treatment $M$ clinicians try to cover $O$ at most 2 of the bacteria giving rise to pneumonia

$$P(O \mid C_1, \ldots, C_n, M)$$
Overall Susceptibility

\[ P_{\tau_k}(o|C_1, \ldots, C_n, M) = \sum_{k \leq l \leq n} \sum_{e_l(S_1, \ldots, S_n)} \prod_{j=1}^{n} P(S_j | C_j, M) \]

- \( C_j \): causal factor \( j \)
- \( S_j \): susceptibility to medication
- \( M \): treatment by antimicrobial medication
- \( O \): overall outcome
Various Models

Conditional probability distributions: \( P(S_j \mid C_j, M) \)

- **susceptibility I model:**
  \[
  P(s_j \mid C_j, M) = \begin{cases} 
  0 & \text{if } C_j = \text{yes}, M = \text{no} \\
  1 & \text{otherwise}
  \end{cases}
  \]

- **susceptibility II model:**
  \( P(s_i \mid \neg c_i, \neg m) = 1 \),
  whereas \( P(s_i \mid \neg c_i, m) = 0 \)

- **susceptibility III model:**
  \[
  P(s_j \mid C_j, M) = \begin{cases} 
  1 & \text{if } C_j = \text{yes, } M = \text{yes} \\
  0 & \text{otherwise}
  \end{cases}
  \]
Model I, Colonised by 1
Model II, Colonised by 1
Model III, Colonised by 1
Model II, Colonised by 2
Property

Let $P(E \mid C_1, \ldots, C_n)$ be defined in terms of the Boolean threshold function $\tau_k$ using the parameters $P(I_k \mid C_k)$, then:

**Theorem:** For each $k$, $0 \leq k \leq n - 1$:

$$P_{\tau_k}(e \mid C_1, \ldots, C_n) \geq P_{\tau_{k+1}}(e \mid C_1, \ldots, C_n)$$

**Proof:**

$$P_{\tau_k}(e \mid C_1, \ldots, C_n) + \sum_{e_{k+1}(I_1,\ldots,I_n)} \prod_{j=1}^{n} P(I_j \mid C_j) = P_{\tau_{k+1}}(e \mid C_1, \ldots, C_n), \text{ and}$$

$$\sum_{e_{k+1}(I_1,\ldots,I_n)} \prod_{j=1}^{n} P(I_j \mid C_j) \geq 0$$
Predicting Optimal Treatment

153 patients with VAP using the \((S_{\text{III}}, k = 1)\) model

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Predicting Optimal Treatment

153 patients with VAP using the \((\text{SIII, } k = 2)\) model

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Image Interpretation

- national breast cancer screening programme
- decision-making under uncertainty
- interpretation of image features in terms of probabilistic graphical models
- from single- to multi-view interpretation
Region features: contrast, size, location, margin, spiculation, etc.

Advantage: a good detection rate per image

Shortcoming: unsatisfactory performance at a patient level because views are treated independently
Multiview Interpretation

Mediolateral oblique view

Craniocaudal view

View–A

View–B
Multiview Bayesian Network

\[
A_i / B_j = (x_1, x_2, ..., x_n)
\]

\begin{itemize}
  \item Interpretation of regions of interest (real-valued feature vector): logistic regression
  \item Combination of region and view information: causal independence
\end{itemize}
Conclusions

Use of modelling approach:

- Select the right qualitative pattern
- Select the right Boolean interaction function
- Fill in arc probabilities $P(I_k | C_k)$

Some future work:

- Study learning of interaction functions from data
- Study other interaction patterns